FastTrack - MA109

Evaluating Expressions and Properties of Real Numbers

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Outline

Attendance Question

Do you have a laptop? Please check the appropriate column on the class list that is being passed around. If you are not on the list, add your name at the bottom and check the appropriate box.

Clicker Question

Do you have a REEF account?
A) YES
B) no

Section 1

Some Review

Natural Numbers $\mathbb{N} = \{1, 2, 3, 4, 5, 6, ...\}$
Whole Numbers $\mathbb{W} = \{0, 1, 2, 3, 4, 5, ...\}$
Integers $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
Rational Numbers $\mathbb{Q} = \{\frac{p}{q} | p, q \in \mathbb{Z}; q \neq 0\}$
Irrational Numbers $\mathbb{H} = \{h | h \notin \mathbb{Q}\}$
Real Numbers $\mathbb{R} = \{\mathbb{Q} \cup \mathbb{H}\}$

Examples: In what sets do the following numbers belong?

1. 7
2. π

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1. 7 $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
2. π $\mathbb{H}, \mathbb{R}$

Absolute Value

Definition

The *absolute value* of n, denoted $|n|$, is the distance of n from 0 on the number line.

Examples: Compute the following.

1. $|5|$
2. $|-3|$
3. $-|-4|$

Absolute Value

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The *absolute value* of n, denoted $|n|$, is the distance of n from 0 on the number line.

Examples: Compute the following.

1. $|5| = 5$
2. $|-3| = 3$
3. $-|-4| =$ -4

Exponents

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

3 is called the base and 4 is called the exponent.

Examples: Compute the following.

1. 2^3
2. $(-1)^5$
3. -3^2

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

3 is called the base and 4 is called the exponent.

Examples: Compute the following.

1. $2^3 = 2 \cdot 2 \cdot 2 = 8$
2. $(-1)^5 = -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 = -1$
3. $-3^2 = -(3 \cdot 3) = -9$

Section 2

Evaluating Expressions

Order of Operations

PEMDAS

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

Parenthesis and Exponents
Multiplication and Division
Addition and Subtraction

Evaluating Expressions

Examples: Evaluate the following expressions using the Order of Operations.

1. $5 + 2 \cdot 3$
2. $8 + 36 \div 4(12 - 3^2)$

Evaluating Expressions

Examples: Evaluate the following expressions using the Order of Operations.

1. $5 + 2 \cdot 3$
   $=5+6$
   $=11$
2. $8 + 36 \div 4(12 - 3^2)$
   $=8 + 36 \div 4(12 - 9)$
   $= 8 + 36 \div 4(3)$
   $= 8 + 9(3)$
   $= 8 + 27$
   $= 35$

Evaluating Expressions

Evaluating a Mathematical Expression

1. Replace each variable with open parentheses ().
2. Substitute the given values for each variable.
3. Simplify using the order of operations.

Example: Evaluate the expression $x^3 - 2x^2 + 5$ **for** $x = -3$**.**

Evaluating Expressions

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1. Replace each variable with open parentheses ().
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Example: Evaluate the expression $x^3 - 2x^2 + 5$ **for** $x = -3$**.**

$(-3)^3 - 2(-3)^2 + 5$

$-27 - 2(9) + 5$

$-27 - 18 + 5$

-40

Section 3

Properties of Real Numbers

Commutativity

The Commutative Properties

Given that a and b represent real numbers:

$$a + b = b + a$$
$$a \cdot b = b \cdot a$$

Terms can be added/multiplied in any order without changing the sum/product.

Associativity

The Associative Properties

Given that a, b, and c represent real numbers:

$$(a+b)+c = a+(b+c)$$
$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Terms can be regrouped.

Identities

The Additive and Multiplicative Identities

Given that x is a real number:

$$x + 0 = x$$
$$1 \cdot x = x$$

Zero is the identity for addition.
One is the identity for multiplication.

The Additive and Multiplicative Inverses

Given that p, q, and x represent real numbers $(p, q \neq 0)$:

$$x + (-x) = 0$$
$$\frac{p}{q} \cdot \frac{q}{p} = 1$$

x and $-x$ are additive inverses.
$\frac{p}{q}$ and $\frac{q}{p}$ are multiplicative inverses.

Distributive Property

The Distributive Property of Multiplication over Addition

Given that a, b, and c represent real numbers :

$$a(b+c) = ab + ac$$
$$ab + ac = a(b+c)$$

Simplifying Algebraic Expressions

Like Terms

$3x^2$ $\frac{-1}{7}x^2$

Non-Like Terms

$5x^3$ $5x^2$

To simplify expressions, we will combine like terms using the Properties of Real Numbers.

Simplifying Algebraic Expressions

To Simplify an Expression

1. Eliminate parentheses by applying the distributive property.
2. Use the commutative and associative properties to group like terms.
3. Use the distributive property to combine like terms.

Example: Simplify the expression completely: $7(2p^2 + 1) - (p^2 + 3)$**.**

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Example: Simplify the expression completely: $7(2p^2+1)-(p^2+3)$**.**

$$\begin{aligned} 7(2p^2+1)-(p^2+3) &= 14p^2+7-1p^2-3 \\ &= (14p^2-1p^2)+(7-3) \\ &= (14-1)p^2+4 \\ &= 13p^2+4 \end{aligned}$$

Section 4

Practice

True or False?

1. $\mathbb{N} \subset \mathbb{W}$
2. $\mathbb{Q} \subset \mathbb{Z}$

Evaluate.

1. $\left|\frac{1}{2}\right|$
2. $-|-4|$

Evaluate using the order of operations.

1. $12 - 10 \div 2 \times 5 + (-3)^2$
2. $\frac{5(-6)-3^2}{9-\sqrt{64}}$

REEF Question

Evaluate using the order of operations.

1. $12 - 10 \div 2 \times 5 + (-3)^2$

Solutions

True or False?

1. $\mathbb{N} \subset \mathbb{W}$ True
2. $\mathbb{Q} \subset \mathbb{Z}$ False, not all rational numbers are integers.

Evaluate.

1. $|\frac{1}{2}|$ $\frac{1}{2}$
2. $-|-4|$ -4

Evaluate using the order of operations.

1. $12 - 10 \div 2 \times 5 + (-3)^2$ -4
2. $\frac{5(-6)-3^2}{9-\sqrt{64}}$ -39

Evaluate for $x = 2$ and $y = -3$

1. $4x - 2y$
2. $6xy^2$

Simply the expression.

1. $3(a^2 + 3a) - (5a^2 + 7a)$
2. $\frac{3}{5}(5n - 4) + \frac{5}{8}(n + 16)$

Solutions

Evaluate for $x = 2$ and $y = -3$

1. $4x - 2y$ $\quad 14$
2. $6xy^2$ $\quad 108$

Simply the expression.

1. $3(a^2 + 3a) - (5a^2 + 7a)$ $\quad -2a^2 + 2a$
2. $\frac{3}{5}(5n - 4) + \frac{5}{8}(n + 16)$ $\quad \frac{29}{8}n + \frac{38}{5}$